

Joint Inversion of Surface-Wave Dispersions and Receiver Functions Based on Deep Learning

Feiyi Wang¹ , Xiaodong Song^{*1,2}, and Jiangtao Li³ 

Abstract

We proposed a deep learning (DL) method to derive V_S models from joint inversion of Rayleigh-wave dispersions and receiver functions, which is based on multilabel convolutional neural network and recurrent neural network. We used a spline-based approach to generate synthetic models instead of directly using existing models to build the training data set, which improves the generalization of the method. Unlike the traditional methods, which usually set a fixed V_P/V_S ratio, our method makes full use of the powerful data mining ability of DL to invert the V_S models assuming different V_P/V_S ratios. A loss function is specially designed that focuses on key features of the model space, for example, the shape and depth of Moho. Synthetic tests demonstrate that the proposed method is accurate and fast. Application to the southeast margin of the Tibetan Plateau shows results consistent with the previous joint inversion with P constraints, indicating the proposed method is reliable and robust.

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Supplemental Material

Introduction

The teleseismic P -wave receiver function (RF; Langston, 1979) is a powerful tool in the studies of Earth's interior, particularly the discontinuities of velocity structure and velocity contrast (Shen *et al.*, 1998; Zhu and Kanamori, 2000). The RF can be obtained by iterative deconvolution of the radial or transverse component and vertical component of the seismogram (Ligorria and Ammon, 1999). Owens *et al.* (1984) extended the RF approach to a wide range of frequency, and used the waveform-fitting method of the teleseismic P -wave RF to invert the crustal velocity, and the linearized inversion based on the teleseismic RF is developed (Owens *et al.*, 1984). Seismic velocity inversion of surface-wave dispersions is also a widely used method to image Earth structures (Shen and Ritzwoller, 2016; Wang *et al.*, 2017, 2023; Cheng *et al.*, 2019; Li *et al.*, 2022). Surface wave is sensitive to vertical shear-velocity averages but insensitive to velocity discontinuities. Thus, joint inversion of surface waves and RFs has become widely used to image Earth's structures to reduce the ambiguity of inversion results (Özalaybey *et al.*, 1997; Julia *et al.*, 2000; Shen *et al.*, 2013; Xu *et al.*, 2013; Sun *et al.*, 2014; Li *et al.*, 2017).

Traditional methods for joint inversion can be categorized into linearization and nonlinearization schemes. Linearized inversions may be limited by the initial models and local extrema. Utilizing alternative cost functions has the potential to enhance the convexity of the misfit space, thereby improving the effectiveness of linearized inversions (Hedjazian *et al.*, 2019).

On the other hand, nonlinear search methods can find the global minimum and do not depend on initial models, but they are often computationally expensive (Bao *et al.*, 2015; Li *et al.*, 2017; Zhang and Yao, 2017; Zhao *et al.*, 2020). In addition, users of a traditional method must choose parameters such as weights of smoothing and damping, often ad hoc, and often subjectively, which will cause the experience of different experts to interfere with the results. Given the aforementioned limitations of the traditional methods for joint inversion, it seems desirable to explore other possible approaches to imaging the velocity structure by surface-wave dispersions and RFs.

In recent years, machine learning, particularly deep learning (DL), has become increasingly prevalent in seismology. Its application includes phases picking (Ross and Ben-Zion, 2014; Wang *et al.*, 2019; Zhang *et al.*, 2020; Zhou *et al.*, 2021; Zhu *et al.*, 2023), event detection (Mousavi *et al.*, 2020; Duan *et al.*, 2023), phase association (McBrearty *et al.*, 2019; Ross *et al.*, 2019), and inversion (Meier *et al.*, 2007; Bianco and Gerstoft, 2018; Cheng *et al.*, 2019; Hu *et al.*, 2020; Cai *et al.*, 2022; Wang

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et al., 2022, 2023). *Chen et al.* (2023) quantitatively evaluated dispersion curve quality by training a neural network model. More comprehensive review of machine learning in seismology can be found in *Bergen et al.* (2019).

In this study, we introduced a DL-based joint inversion of RFs and surface waves to derive V_S models. By constructing a reasonable training data set and leveraging the powerful non-linear mapping ability of DL, we enhanced the generalization (i.e., applicability) of the method and the accuracy of the inverted V_S models for different V_P/V_S ratios. The V_P/V_S ratio is generally not known and is normally set to the global average (1.75) in conventional joint inversions (*Shen et al.*, 2013; *Xu et al.*, 2013; *Sun et al.*, 2014; *Li et al.*, 2017). We verified the method with synthetic tests with complex constructed models and real data used previously in a joint version with additional V_P constraints (*Li et al.*, 2017). The joint inversion (*Li et al.*, 2017) has better estimated V_P/V_S ratios, allowing direct comparisons with this study. In addition to V_S , we also obtain V_P . However, because the data we used are mainly sensitive to V_S , and the synthetic data tests also demonstrate that the error of V_S is smaller, the obtained V_P is for reference only.

Methods and Data

Architecture and loss function of the SRF-Net

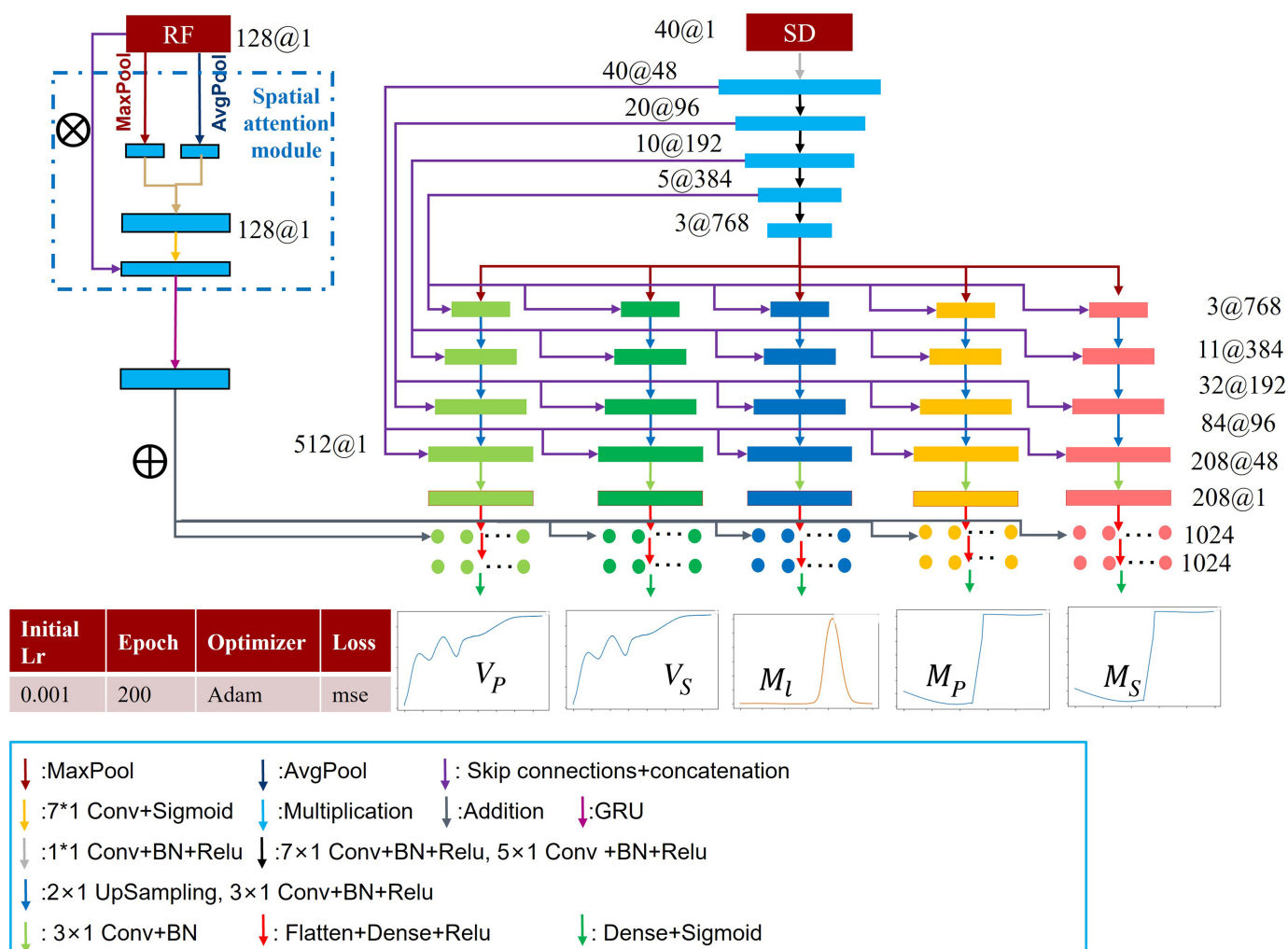
The network used in our study, named SRF-Net (standing for a network of Surface wave and Receiver Function), was specifically designed according to the characteristics of dispersions and RFs. We treated the Rayleigh-wave phase and group velocity dispersion curves as “1D images” and applied a convolutional neural network (CNN) to extract the intrinsic information. Our CNN structure was based on U-Net and ResNet (*Ronneberger et al.*, 2015; *He et al.*, 2016). As for RFs, because they are essentially time series, we utilized a gate recurrent unit (GRU) network, which is a type of recurrent neural network known for its excellent performance in processing time series (*Cho et al.*, 2014). In addition, we incorporated a spatial attention module before the GRU layer to account for the varying importance of waveforms (for instance, the P_s phase is generally considered to be more important) in different time windows of the RFs (*Woo et al.*, 2018). We show the distribution of spatial attention weights using a synthetic data example in Figure S1, available in the supplemental material to this article. It is evident that the network exhibits larger weights on the P_s phase, the magnitude of which exceeds that of the RF itself. This observation emphasizes the importance that the network places on extracting information from the P_s phase. In addition, considering the higher energy of the direct P phase, the network also exhibits larger weights near the direct P phase. To combine the outputs of the two networks, we employed a fully connected layer. Figure 1 displays the architecture of SRF-Net, the acronyms and systems adhere to the conventions of DL, and all of the acronyms used in this article are listed in Table S1 (*Ronneberger et al.*, 2015; *Raschka and Mirjalili*, 2019; *Yang and Ma*, 2019).

$$\text{Loss} = \lambda_1 \|V_P^{\text{pred}} - V_P^{\text{true}}\|_2^2 + \lambda_1 \|V_S^{\text{pred}} - V_S^{\text{true}}\|_2^2 + \lambda_2 \|M_P^{\text{pred}} - M_P^{\text{true}}\|_2^2 + \lambda_2 \|M_S^{\text{pred}} - M_S^{\text{true}}\|_2^2 + \lambda_3 \|M_I^{\text{pred}} - M_I^{\text{true}}\|_2^2. \quad (1)$$

We developed a multilabel model with a specific loss function (equation 1) that takes into account the distinct surface-wave and RF characteristics in the inversion of velocity structures. In the loss function, the first and second terms are for the overall profile of the velocity structures. The V_P^{pred} and V_S^{pred} indicate the predicted V_P and V_S of the SRF-Net, respectively. The V_P^{true} and V_S^{true} indicate the labels of the training data set. Theoretically, these two terms contain all the information about the velocities to be inverted. However, they are insufficient because some details such as the shape and depth of the Moho, which influence the RFs greatly, may be not inverted well. Thus, we added several additional Moho terms in the loss function to focus on the Moho. The third and fourth terms (M_P and M_S) are used to constrain the shape of the Moho for P - and S -wave velocities, respectively. We separated the V_P and V_S models of 10 km above and below the Moho depth as the labels. The Moho depth is defined in the [Construction of the training data set](#) section. The final term constrains the location of the Moho depth using a Gaussian distribution. This Moho location curve of a single V_S model (M_I) contains 600 points, and the peak (location parameter μ in Gaussian distribution) represents the Moho depth value. The standard deviation (size parameter σ in Gaussian distribution) is 50 points (*Wang et al.*, 2022). The final predicted V_P and V_S velocities are obtained by substituting the Moho shape terms for the overall trend terms at the predicted Moho depth. Discontinuities may be introduced when substituting the Moho into the overall velocity profile. To suppress these tiny discontinuities, we set a mean filter with a window length of 3 to smooth the upper and lower connections (equations 2 and 3). In summary, our inputs are two 1D vectors of length 128 and 40, the first vector of length 128 represents the RF of length 12 s, and the second vector represents the phase and group velocity values on corresponding 20 periods. The outputs are five 1D vectors, which are V_P and V_S overall velocity of length 1500, V_P and V_S velocity near Moho of length 200, and a vector of length 600 representing Moho depth. We experimented different weights (Table S2) and found that $\lambda_1 = 0.3$, $\lambda_2 = 0.2$, and $\lambda_3 = 0.1$ yielded the best performance. The different weights used in Table S2 perform similarly, indicating that the structure of the SRF-Net is reliable.

$$V_i = \sum_{i=M-150}^{M-50} \frac{V_{i-1} + V_i + V_{i+1}}{3}, \quad (2)$$

$$V_i = \sum_{i=M+50}^{M+150} \frac{V_{i-1} + V_i + V_{i+1}}{3}, \quad (3)$$



in which V_i is the velocity values after substituting the Moho shape terms for the overall trend terms at the predicted Moho depth. The M is the Moho depth predicted by the network.

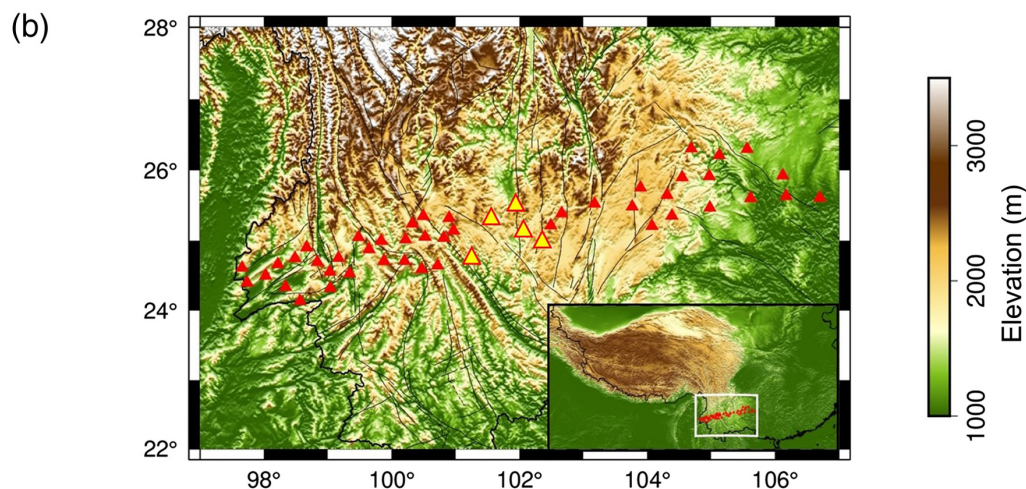
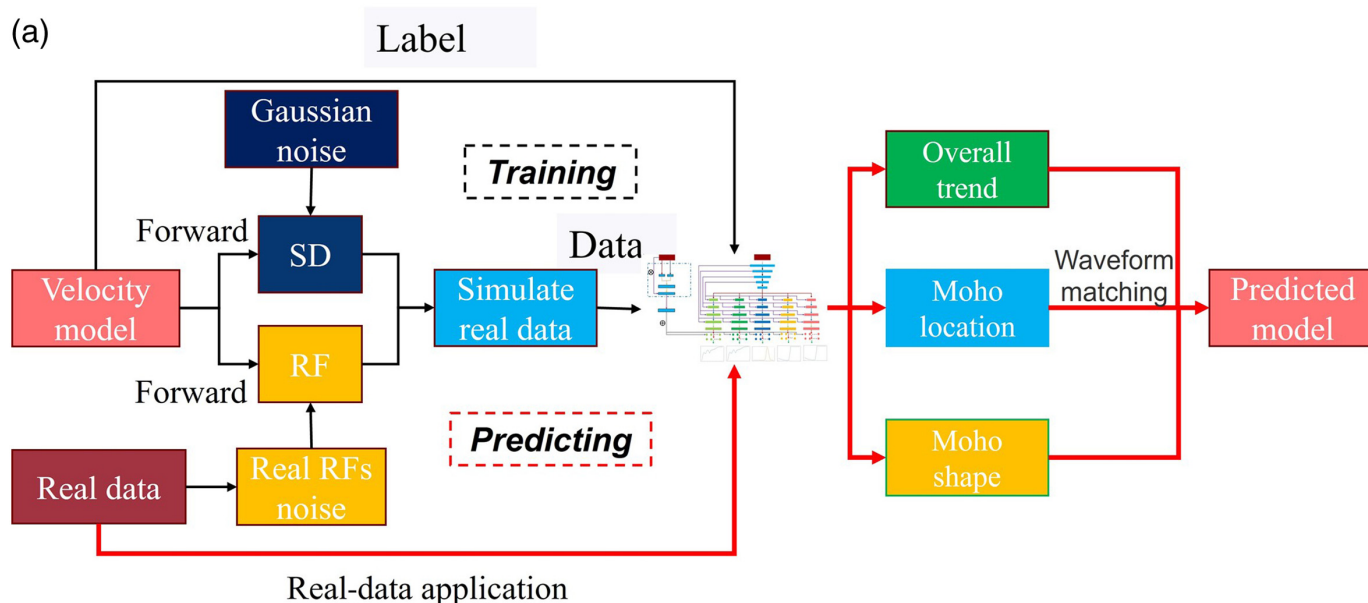
Construction of the training data set

The quality of the training data set is crucial to the performance of DL methods (Deng *et al.*, 2009). To improve the diversity of the training data set, we used a spline parameterization method modified from Li *et al.* (2017) to obtain smooth velocity-depth profiles. Each velocity pair (V_S and V_P) is divided into three parts, that is, sedimentary layer, crust, and mantle. We used a linear gradient in V_S and allowed for variations of the thickness of the sedimentary layer as well as the top and bottom velocities. In the crust and mantle, V_S is controlled by two sets of cubic splines. The number of crustal spline nodes ranged randomly from 5 to 10 from a uniform distribution, whereas the number of mantle spline nodes remained fixed at 7. The last node of the crustal spline has the same value as the first node of the mantle spline, which is called the connecting node. The depth of the connecting node is defined as the Moho depth, which is randomly set between 25 and 75 km from a uniform distribution. We set uniform node spacing in the crust but a

Figure 1. Network used in this study. All of the acronyms used in the figure are listed in Table S1. The color version of this figure is available only in the electronic edition.

stretching spline to control the node spacing in the mantle to simulate sharp and gradual Moho situations. The ranges of values of V_S are 1.5–2.9, 2.9–4, and 4–5 km/s in the sedimentary layer, crust, and mantle, respectively. In the crust, the V_P/V_S ratio was randomized between 1.6 and 1.9. All variable parameters mentioned earlier are randomly generated within the corresponding setting range. The V_P/V_S ratio is fixed at 2.0 (Shen *et al.*, 2013) in the sedimentary layer and 1.8 in the mantle, which were used in Li *et al.* (2017). The density was calculated by the Brocher relationship (equation 4; Brocher, 2005). We discretized each 1D velocity model into 1500 layers, each with a uniform thickness of 0.1 km. The model space of the training data set is shown in Figure S1.

$$\rho = 1.6612V_P - 0.4721V_P^2 + 0.0671V_P^3 - 0.0043V_P^4 + 0.000106V_P^5. \quad (4)$$



We used the Computer Programs in Seismology (Herrmann, 2013) to calculate Rayleigh-wave phase and group velocity dispersion curves at a period of 10–70 s and RFs. To simulate real data and enhance the generalization of our DL model, we added Gaussian random noise to the theoretical dispersion curves, with a standard deviation σ of 0.015 km/s for periods of 10–20 s, 0.02 km/s for 20–35 s, 0.03 km/s for 35–45 s, and 0.05 km/s for 45–70 s. The noise amplitudes were guided by Shen and Ritzwoller (2016). For the noise of RFs, we adopted the approach used in Wang et al. (2022). We use the IASP91 model (Kennett and Engdahl, 1991) to moveout the RFs with a reference slowness of 6.4 s/deg (Yuan et al., 1997). Figure 2a shows the workflow of the proposed method.

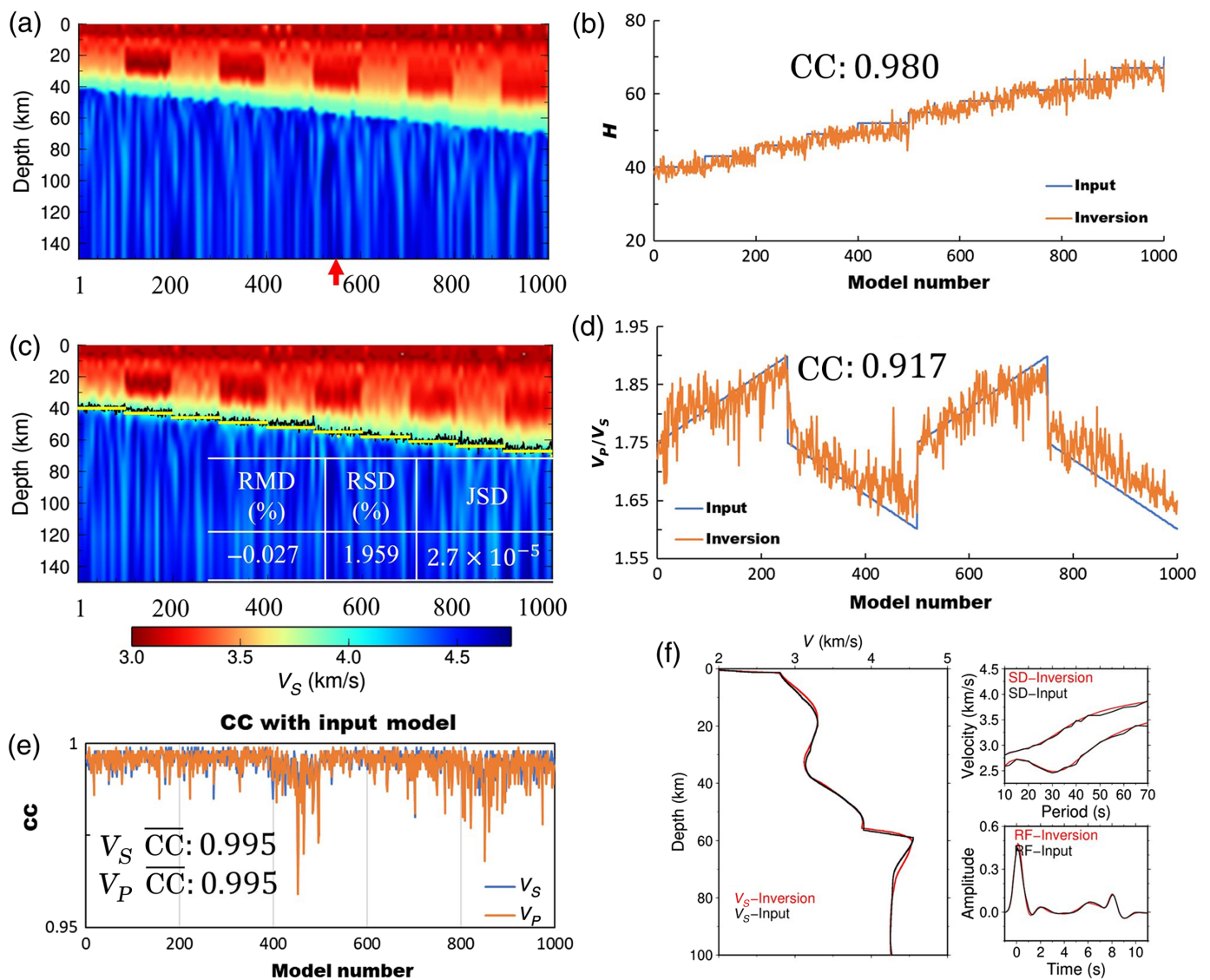
We generated 80,000 input–output pairs as the training data set and 20,000 pairs as the validation data set. The validation data set is excluded from the training process and only used to guide parameter tuning and to avoid overfitting. We conducted 200 training epochs and performed validation after each epoch to monitor the training effect. We used open-source libraries

Figure 2. (a) Workflow of the proposed method and (b) study region. The red triangles are the broadband seismic stations used in this study. Inset in panel (b) shows the location of the study region in southwest China. The station array was used in Li et al. (2017) for a joint inversion. The yellow triangles represent the stations shown in Figure 6. The color version of this figure is available only in the electronic edition.

TensorFlow and Keras to support all of the aforementioned DL concepts and optimization algorithms. All computations were performed on two NVIDIA A100 graphic processing units with 40 GB memory. The total training time was approximately 6 hr. Loss functions during the training process are shown in Figure S3.

Synthetic tests and application to real data

We used both synthetic and real data to evaluate the performance of SRF-Net. For the synthetic data, the input models include 1000 1D velocity pairs, which were used to generate

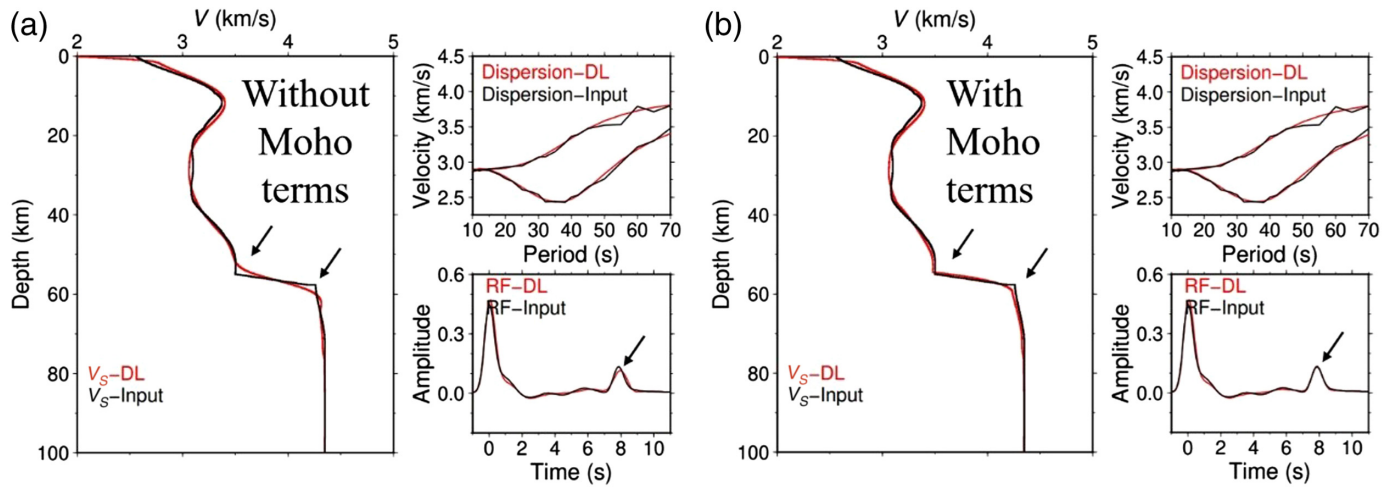


dispersion curves and RFs as the input to the SRF-Net. A single example is shown in Figure 3f, and the whole model space is shown in Figure S2. The synthetic models encompassed various geological structures. The V_p/V_s ratio gradually increases from 1.75 to 1.9 in the 1st to 250th and 501st to 750th models, while decreasing from 1.75 to 1.6 in the 251st to 500th and 751st to 1000th models. The input models are shown in Figures 3a. We added noise to these synthetic models in the same way we constructed the training data set (see the [Construction of the training data set](#) section) to the forward calculations.

In addition to synthetic data tests, we also applied our method to real data. The real data are from a linear broadband seismic array in the southeast margin of the Tibetan Plateau (Fig. 2b). The data were used previously by Li *et al.* (2017) in a joint inversion of RFs and surface-wave dispersion with additional P constraints, which resulted in better-constrained models of V_s in the crust and mantle, V_p/V_s ratio in the crust, and Moho depth. Li *et al.* (2017) include P -wave RFs (of the P_s

Figure 3. Results of synthetic tests. (a) Input V_s models. The red arrow marks the selected model used to show the 1D results in panel (f). (b) The input and inverted Moho depths. The CC of predicted and input Moho depths is labeled in the panel. (c) Inverted V_s models. The black and yellow lines represent the predicted and input Moho depths, respectively. The relative mean deviation (RMD), relative standard deviation (RSD), and Jensen–Shannon divergence (JSD) are defined in the equations (6), (7), and (9). (d) The V_p/V_s ratio trends of the inverted and input models. The CC of predicted and input V_p/V_s ratio trends is labeled in the panel. (e) Cross correlation (CC) between the inverted and input models. The CC is defined in equation (5) and the CC is the average of CC of the 1000 models. (f) An example of 1D model inversion located at the red arrow in (a). These examples demonstrate the effectiveness of the proposed method. The color version of this figure is available only in the electronic edition.

converted phase) recorded by 50 temporary broadband stations (Fig. 2b) and Rayleigh-wave phase and group dispersions at periods of 10–70 s. In the processing of RFs, the Gaussian width used was 2.5, and we cut the RFs to 12 s to limit the



crustal multiples. The total number of sampling points of phase and group velocity dispersions was 40 at 10, 12, 15, 18, 20, 22, 25, 28, 30, 32, 35, 38, 40, 42, 45, 50, 55, 60, 65, and 70 s. Details regarding data processing are provided in Li et al. (2017).

Result

Synthetic tests

We used synthetic models to test the performance of the SRF-Net. The test models contain low-velocity sedimentary layers and discontinuous low-velocity blocks in the crust (by setting values of nodes on corresponding depths; Fig. 3a). The depth of the Moho increases from an initial 45 km to a maximum of 70 km. The velocities in the mantle are generated without any additional constraints in the same way as the training set.

Figure 3 shows the results of the synthetic tests. The pattern of the inverted model (Fig. 3b) matches that of the input model (Fig. 3a) quite well. We can measure the agreement between the input and inverted models using the following parameters:

The cross correlation (CC):

$$CC = \frac{\sum_{n=1}^N [(x_n - \bar{x}) * (y_n - \bar{y})]}{\sqrt{\sum_{n=1}^N (x_n - \bar{x})^2 \sum_{n=1}^N (y_n - \bar{y})^2}}. \quad (5)$$

The relative mean deviation (RMD):

$$RMD = \frac{\sum_{i=1}^L \sum_{n=1}^N (x_{ni} - y_{ni}) / y_{ni}}{NL}. \quad (6)$$

The relative standard deviation (RSD):

$$RSD = \left\{ \frac{\sum_{i=1}^L \sum_{n=1}^N [(x_{ni} - y_{ni}) / y_{ni}]^2}{NL} \right\}^{1/2}. \quad (7)$$

The Jensen–Shannon divergence (JSD):

$$KLD(x||y) = \sum_{i=1}^L \sum_{n=1}^N x_{ni} \log \frac{x_{ni}}{y_{ni}}, \quad (8)$$

Figure 4. The comparison of results with and without Moho terms. (a) The result without the Moho terms. (b) The result with the Moho terms. Pay attention to the results with the Moho terms finely capture details near the Moho, and the receiver function (RF) calculated by the inverted models exhibits a better match with the input RF (depicted by black arrows). The color version of this figure is available only in the electronic edition.

$$JSD(x||y) = \frac{1}{2} KLD\left(x||\frac{x+y}{2}\right) + \frac{1}{2} KLD\left(y||\frac{x+y}{2}\right), \quad (9)$$

in which x_{ni} and y_{ni} are the i th values of the n th input and inverted models, respectively. The x and y are the average of x_{ni} and y_{ni} , respectively. The KLD is Kullback–Leibler divergence.

The inverted velocity models exhibit high CC and low JSD with the input models (Fig. 3c). The CC for an individual grid point may vary, but they are almost all greater than 0.97, with an average CC of 0.995 for both V_S and V_P (Fig. 3c). The predicted Moho depth is also consistent with that of the input models with the CC of 0.980 between input and inverted models (Fig. 3d). Our method is capable of capturing the detailed features of the input model, as demonstrated by the RMD, RSD, CC, and JSD analyses (labeled in each panel).

The red arrow in Figure 3a marks the location of the 1D V_S model used as an example to show the details of results. The selected model is representative and contains a thick sedimentary layer, a distinct low-velocity layer in the crust, and a sharp Moho. Figure 3f shows that the SRF-Net inversion results recovered the aforementioned features almost exactly, and the predicted surface-wave dispersion curves and RF obtained by forward modeling based on the inversion model exhibit a good agreement with the theoretical data. The impact of Moho terms (i.e., the third, fourth, and last terms in equation 1) on the inversion results is shown in Figure 4. Specifically, the velocity models near the Moho interface exhibit finer details when Moho terms are included in the inversion process,

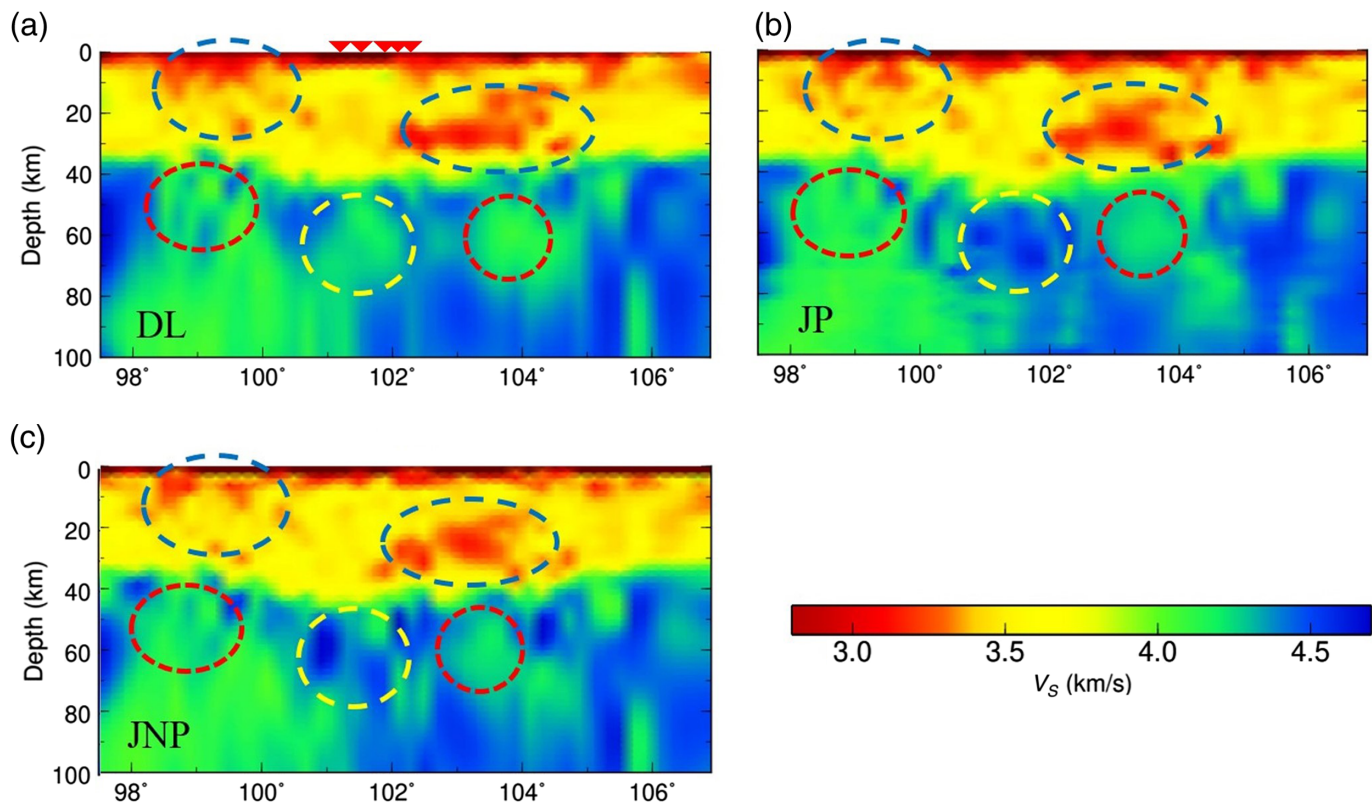


Figure 5. Comparison of the results of deep learning (DL) and previous results JP (Li *et al.*, 2017). (a) The result from DL. (b) The JP from Li *et al.* (2017). (c) The JNP (Joint inversion without V_P models, see below) from Li *et al.* (2017). The red triangles in panel (a) mark the location of the stations in Figure 6. The blue and red circles mark the low velocity zone (LVZ) in the crust and mantle, respectively. The yellow circles mark the locations of the biggest difference between the DL result with the JP and JNP. The color version of this figure is available only in the electronic edition.

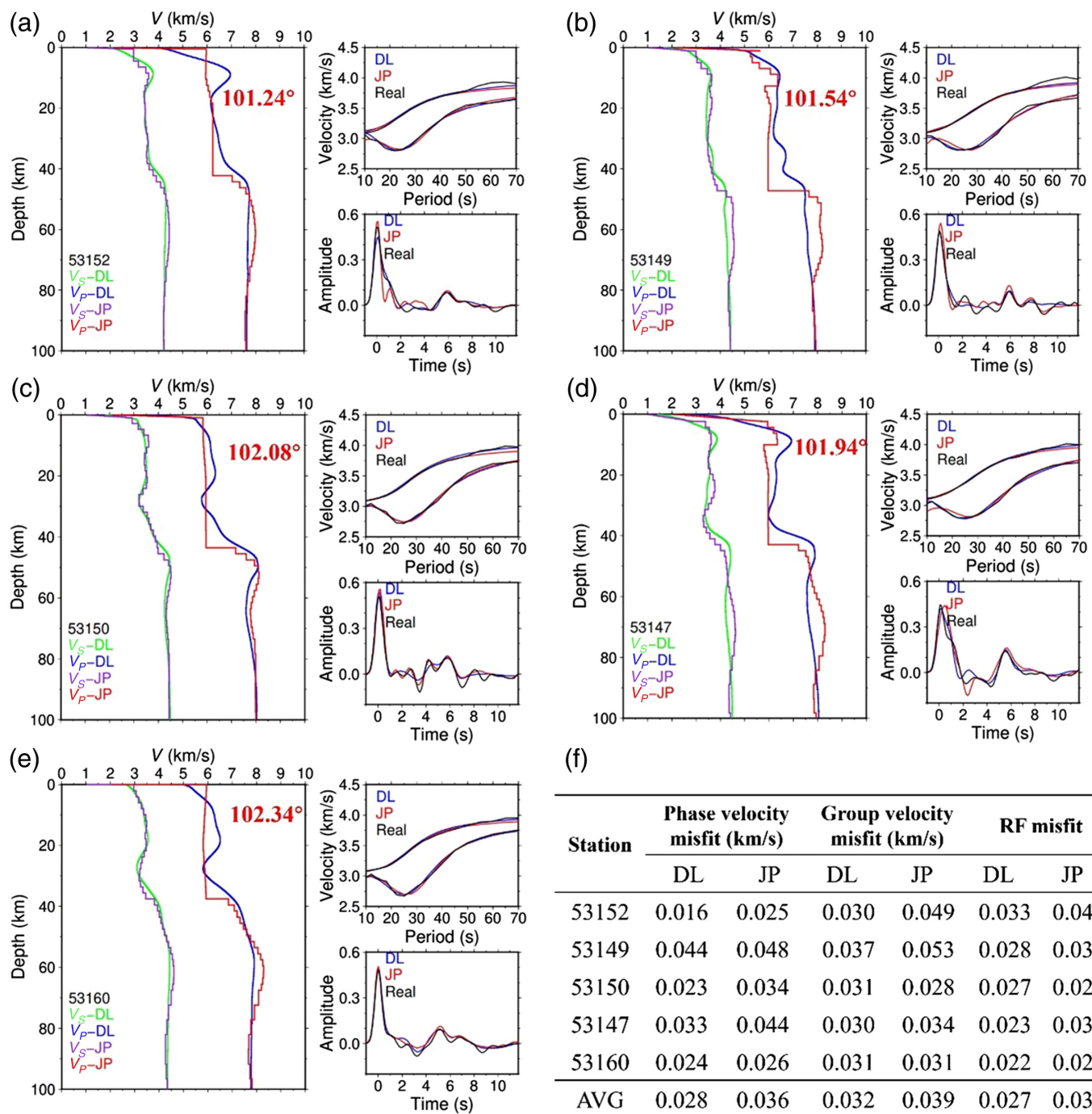
and the RF calculated by the inverted models exhibits better agreement with the input RF compared to the results obtained without Moho terms (black arrows in Figure 4a,b). Because of the “black box” nature of DL, the uncertainties of the recovered models are difficult to estimate theoretically (Lakshminarayanan *et al.*, 2017; Hu *et al.*, 2020). However, we can obtain some estimates of the uncertainty from the synthetic tests. By counting the inversion errors of all sampling points, we determined that the RMD and RSD of V_S models are shown in Figure 3b. The time required to predict 1000 pairs of 1D V_P and V_S models using the trained SRF-Net is ~ 20 s, which means that predicting each pair of models takes ~ 0.02 s. This is many times faster than traditional methods (Hu *et al.*, 2020; Wang *et al.*, 2021; Mousavi and Beroza, 2022). Overall, the synthetic tests demonstrate the effectiveness and efficiency of the proposed method.

Application to real data

We applied SRF-Net to the real data, which was used by Li *et al.* (2017) to obtain V_S models (hereafter referred to as JP). Unlike the conventional joint inversion of surface-wave dispersion curves and RFs, Li *et al.* (2017) added V_P constraint to the joint inversion to improve the performance. We compared our inversion models with the JP, and the results are shown in Figure 5.

Figure 5 shows that the DL result is generally consistent with the JP. Figure 5a,b shows that both profiles have two distinct low velocity zone (LVZ) in the crust (blue circles in Fig. 5a,b) and mantle (red circles in Fig. 5a,b). The Moho

trends are also similar, increasing from 35 km at 97° E to the deepest at 101° E, and then gradually decreasing to ~ 40 km at 107° E. There are certain differences between the two results in details, and the biggest difference is the structure in the shallow mantle at 101°–102° E (yellow circles in Fig. 5a, b). We compared the corresponding station data and found that the RFs are too complex to be well fitted by both inversion models (Fig. 6). However, our fitting errors are smaller than JP (reduced phase, group velocity dispersions, and RFs average misfits by 22%, 18%, and 17%, respectively), indicating that our results may be more reliable. We also calculated the CC of the two inversion models. The CC of almost all stations is above 0.9, and the average CC is 0.960, which show that the DL result is very similar to the JP. Finally, we compared all the dispersions and RFs misfits of the two inversion models with the observations and calculated the misfit of the data by root mean square (rms) error (equation 10). The results (Figs. S4, S5) show that there is almost no difference. Unlike other



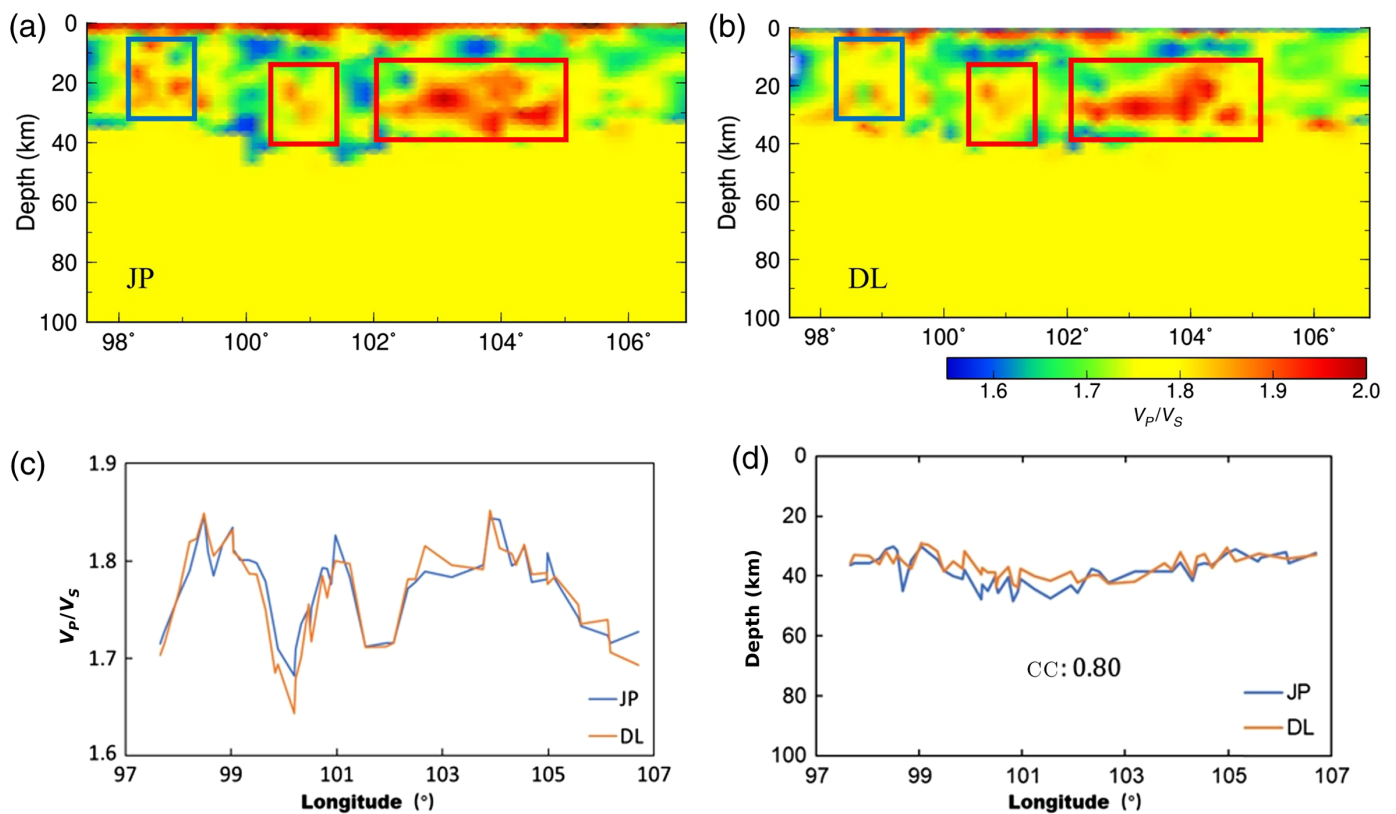
DL methods used for inversion in seismology (Hu *et al.*, 2020; Cai *et al.*, 2022; Yang *et al.*, 2022), we did not use any existing velocity models to construct the training data set. The synthetic tests in the Synthetic tests section and real-data application imply that our trained SRF-Net has high generalization. Our method is based on the inversion of a single station, so theoretically if the stations are dense enough, the 3D velocity structure can also be inverted.

The rms error:

$$\text{Rms} = \left\{ \frac{\sum_{i=1}^N [(x_i - y_i)]^2}{N} \right\}^{1/2}, \quad (10)$$

Figure 6. The results of stations at 101°–102° E (yellow circles in Fig. 5a,b). The results of the station at (a) 101.24° E, (b) 101.54° E, (c) 102.08° E, (d) 101.94° E, (e) 102.34° E. (f) Statistical results for five stations. AVG means average misfit of these five stations. From these comparisons, it can be seen that the misfit of DL is smaller. The color version of this figure is available only in the electronic edition.

in which x_i and y_i are the i th values of the calculated data (surface-wave dispersions or RFs) from forward modeling based on the inverted models and the observations, respectively.



Discussion

Comparison of different inversions

The model JP discussed earlier was obtained by the joint inversion of RFs and dispersions with the constraint of V_p models (Li *et al.*, 2017). However, we used RFs and dispersions only in our study. For a more comprehensive assessment, we compared our models with the result of conventional joint inversion using RFs and dispersion curves only (hereafter referred to as JNP). The JNP was also obtained from Li *et al.* (2017) by fixing the V_p/V_s to 1.75.

Figure 5a–c shows the results from different inversions. The three profiles show consistent patterns as well as certain differences. The three profiles contain two main LVZs in the crust and similar Moho trends. We calculated the CC and rms of our results with JNP and JP. The CC of our result with the JP (0.960) is larger than that with the JNP (0.926) (the CC of JP and JNP is 0.946), and the rms is smaller with the JP (0.137 km/s) than that with the JNP (0.183 km/s) (the rms of JP and JNP is 0.154 km/s). The earlier comparison shows that compared to JNP, our result is closer to the JP, which is considered to be better in Li *et al.* (2017). The reason for the result of this comparison may be that surface-wave dispersions are also sensitive to V_p (Curtis *et al.*, 1998; Zhou *et al.*, 2006; Ying *et al.*, 2018), and we considered different V_p/V_s ratios when constructing the training data set, and made full use of the powerful nonlinear mapping and data mining capabilities of DL.

Figure 7. The V_p/V_s ratio and Moho depth of JP and DL results. (a) The V_p/V_s ratio of JP. (b) The V_p/V_s ratio of DL results. (c) Average V_p/V_s ratios in the crust at each station. (d) The Moho depths at each station. The red and blue boxes mark the high V_p/V_s ratio in the crust. The CC of JP and DL Moho is labeled in the panel. The color version of this figure is available only in the electronic edition.

Moho depth and V_p/V_s ratio

The Moho depth (H) trades off with the average V_p/V_s ratio (k) in RFs, and these two parameters control the time separation between direct P and P_s phases when the ray parameter is fixed (Zhu and Kanamori, 2000). However, the predicted H and k by SRF-Net are not entirely correct. Thus, when applying SRF-Net to real data, we set small ranges of H (± 2.5 km) and V_p ($\pm 1\%$) based on the predicted values by SRF-Net, and searched for the optimal model that best fits the observations. The V_s was fixed while searching the models because the average V_s can be well constrained by the dispersions. Although this search process may add some extra time, this extra time was completely negligible compared to the times taken by traditional joint inversion methods and the huge amount of data processing.

To further verify the reliability of the V_s obtained from SRF-Net, we calculated the V_p/V_s ratio using the V_p introduced in Li *et al.* (2017). Figure 7 compares the V_p/V_s ratio

Station: 53064

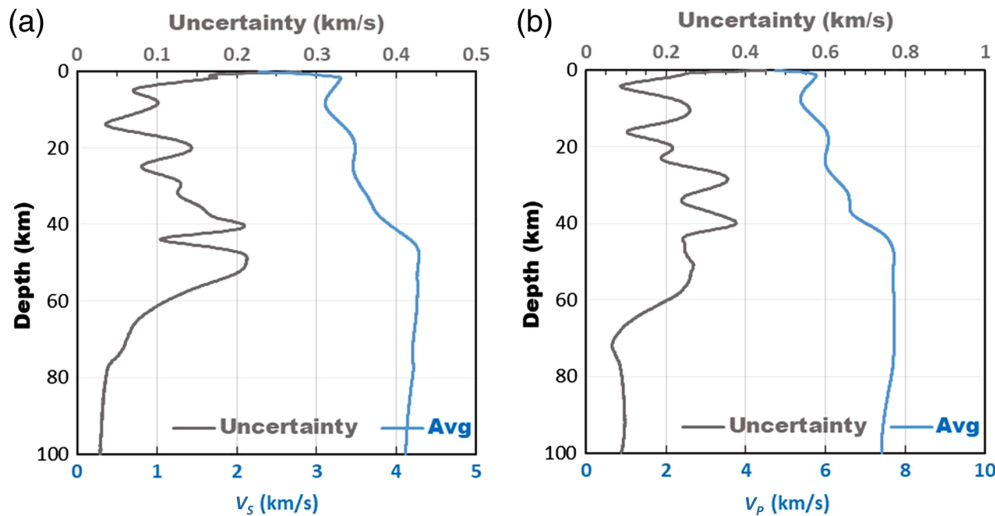


Figure 8. The depth-dependent uncertainties estimated by bootstrap tests on station 53064. (a) Average V_S and its uncertainty. (b) Average V_P and its uncertainty. The color version of this figure is available only in the electronic edition.

profiles from different inversions. There are three distinct high- V_P/V_S ratio regions in the results of JP (red and blue boxes in Fig. 7a), two of which corresponding to the two LVZs in the crust in Figure 5, which suggests possible partial melt (Li *et al.*, 2017). In the results of DL, the high- V_P/V_S ratio in the crust at 98.5° E is not as significant (blue box in Fig. 7b), but the high- V_P/V_S ratio area at 101° and 104° E (red boxes in Fig. 7b) is obvious. These differences are probably because no V_P was introduced in the DL inversion. Figure 7c summarizes the average V_P/V_S ratio over the whole crust for each station. The average V_P/V_S ratio of DL is very similar to that of JP, indicating that our V_S is relatively reliable on the average scale of the whole crust. The Moho depths of two inversions are basically consistent, with a CC of 0.80. The biggest difference is at longitude 101°–102° E, corresponding to the part where the inversion results differ the most in Figure 5a,b (yellow circles). The comparison with the observed data (Fig. 6) demonstrated that our results are more reliable.

Model uncertainty estimation

The uncertainty of the inverted models obtained by DL methods is challenging. Most of the uncertainty estimation algorithms are based on Bayesian neural networks (which can be understood as a probabilistic neural network, for which parameters are considered to obey a certain prior distribution), and when given an input, an output with a certain distribution property can be generated for calculation model uncertainty (Kononenko, 1989). Meier *et al.* (2007) and Earp *et al.* (2020) used mixture density networks and Zhang and Curtis used invertible neural networks to estimate full posterior probability

distribution functions of the output V_S model. Based on the quality of the input data, Cai *et al.* (2022) added noise to the data and used bootstrap tests to estimate the uncertainty of the results.

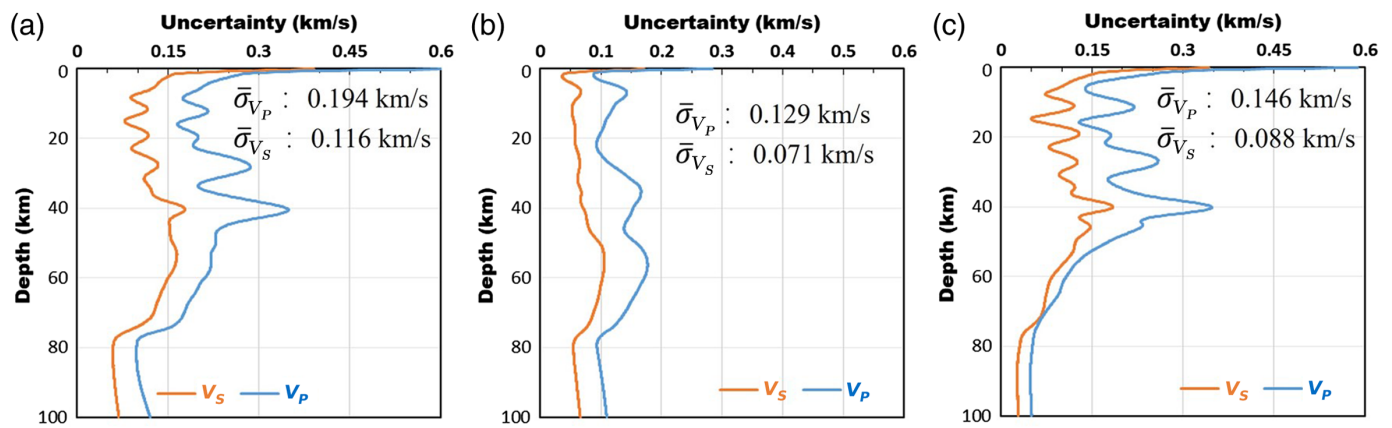
Here, we also adopted the strategy of bootstrap tests to estimate the model uncertainty. For each station, we resample the observations 200 times by adding Gaussian noise to the dispersion curves according to the noise level at each period and to RFs as in the Construction of the training data set section. Considering that the real data already contain noise, compared to the noise added when building the training data set in the Construction of the training

data set section, the noise level we add here is only half. This data set is then input to the trained SRF-Net to predict V_S and V_P models. Finally, we calculate the standard deviation of the 200 V_S and V_P models to estimate the V_S and V_P models uncertainty, respectively.

We take station 53064 as an example (Fig. 8) to show the impact of noise on the inversion and calculate the average results of all data from 50 stations (Fig. 9). Figure 9a shows the result of adding noise both to the dispersion and RFs. Figure 9b shows the result of only adding noise to the dispersion and Figure 9c shows the result of only adding noise to the RFs. All three of the estimated uncertainties are larger in the top 5 km, which is due to the lack of short period data (<10 s). The uncertainty is largest for both adding noise to dispersions and RFs (Fig. 9a). When RFs data are fixed, the uncertainty is smallest (V_S : 0.071 km/s and V_P : 0.129 km/s in Fig. 9b), which probably means SRF-Net is more sensitive to RFs. When adding noise to RFs (Fig. 9c), the uncertainty is concentrated mainly in the crust (<40 km), reflecting the sensitivity of the RFs to the Moho depth. The uncertainty of V_S when only adding noise to dispersion is smaller than that in the similar previous study (Cai *et al.*, 2022), implying that our method is relatively robust. This method of estimating uncertainty is closely related to the level of noise added, with uncertainty increasing as the noise level increases.

Conclusion

In this study, we proposed a DL-based method to invert the V_S , to a certain extent, V_P models from surface waves and RFs. Flexible parameterization of the method to build a training



data set enables SRF-Net to have high-generalization utility. Utilizing the powerful data-mining ability of DL, SRF-Net quickly and efficiently inverted the V_S models under different V_P/V_S ratios.

Synthetic tests demonstrate that the SRF-Net is efficient and accurate. Application to real data shows that the method is robust and feasible. The real data were never used in the construction of the training data set, but consistent results and comparable misfits with observations are obtained compared to the previous inversion. Thanks to the high generalization of SRF-Net, we only need to train the network once to apply it to different regions, which greatly improve the applicability of the network and speeds up the efficiency. For the future studies, we plan to introduce more data (e.g., gravity) in the inversion to reduce model ambiguity. In addition, how to better estimate the uncertainty of the inversion results is also worth studying.

Data and Resources

The receiver functions (RFs) and surface-wave dispersions used in this study were from Li *et al.* (2017, doi: [10.1002/2017JB014135](https://doi.org/10.1002/2017JB014135)). Some figures were plotted with the Generic Mapping Tools (GMT) 4.5.18 software (<https://www.generic-mapping-tools.org/download/>, last accessed April 2024 [Wessel and Smith, 1998]). The data set can be obtained upon request from the authors or directly from Zenodo (doi: [10.5281/zenodo.8126289](https://doi.org/10.5281/zenodo.8126289)). The supplemental material for this article includes the model space of training data set, loss curves, comparing misfit for different methods to observations, and table of different weights used in training the DL model.

Declaration of Competing Interests

The authors acknowledge that there are no conflicts of interest recorded.

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Figure 9. The depth-dependent uncertainties estimated by bootstrap tests. (a) The uncertainty when adding noise to both dispersions and RFs. (b) The uncertainty when only adding noise to dispersions. (c) The uncertainty when only adding noise to RFs. The average uncertainty of all depth is labeled in the panels. The color version of this figure is available only in the electronic edition.

References

- Bao, X., X. Sun, M. Xu, D. W. Eaton, X. Song, L. Wang, and P. Wang (2015). Two crustal low-velocity channels beneath SE Tibet revealed by joint inversion of Rayleigh wave dispersion and receiver functions, *Earth Planet. Sci. Lett.* **415**, 16–24, doi: [10.1016/j.epsl.2015.01.020](https://doi.org/10.1016/j.epsl.2015.01.020).
- Bergen, K. J., P. A. Johnson, M. V. de Hoop, and G. C. Beroza (2019). Machine learning for data-driven discovery in solid Earth geoscience, *Science* **363**, no. 6433, eaau0323, doi: [10.1126/science.aau0323](https://doi.org/10.1126/science.aau0323).
- Bianco, M. J., and P. Gerstoft (2018). Travel time tomography with adaptive dictionaries, *IEEE Trans. Comput. Imaging* **4**, no. 4, 499–511, doi: [10.1109/tci.2018.2862644](https://doi.org/10.1109/tci.2018.2862644).
- Brocher, T. M. (2005). Empirical relations between elastic wavespeeds and density in the earth's crust, *Bull. Seismol. Soc. Am.* **95**, no. 6, 2081–2092, doi: [10.1785/0120050077](https://doi.org/10.1785/0120050077).
- Cai, A., H. Qiu, and F. Niu (2022). Semi-supervised surface wave tomography with Wasserstein cycle-consistent GAN: Method and application on Southern California Plate Boundary Region, *J. Geophys. Res.* **127**, no. 3, e2021JB023598, doi: [10.1002/essoar.10505230.1](https://doi.org/10.1002/essoar.10505230.1).
- Chen, S., R. Gao, Z. Lu, Y. Liang, W. Cai, L. Cao, Z. Chen, and G. Wang (2023). Quality influencing factors of dispersion curves from short period dense arrays based on a convolutional neural network across the north section of the Xiaojiang fault area, *Earthq. Sci.* **36**, no. 3, 200–211, [10.1016/j.jeqs.2023.04.004](https://doi.org/10.1016/j.jeqs.2023.04.004).
- Cheng, X., Q. Liu, P. Li, and Y. Liu (2019). Inverting Rayleigh surface wave velocities for crustal thickness in eastern Tibet and the western Yangtze craton based on deep learning neural networks, nonlinear process, *Geophysics* **26**, no. 2, 61–71, doi: [10.5194/npg-26-61-2019](https://doi.org/10.5194/npg-26-61-2019).
- Cho, K., B. Van Merriënboer, C. Gulcehre, D. Bahdanau, F. Bougares, H. Schwenk, and Y. Bengio (2014). Learning phrase representations using RNN encoder-decoder for statistical machine translation, doi: [10.48550/arXiv.1406.1078](https://doi.org/10.48550/arXiv.1406.1078).

- Curtis, A., J. Trampert, R. Snieder, and B. Dost (1998). Eurasian fundamental mode surface wave phase velocities and their relationship with tectonic structures, *J. Geophys. Res.* **103**, no. B11, 26,919–26,947, doi: [10.1029/98JB00903](https://doi.org/10.1029/98JB00903).
- Deng, J., W. Dong, R. Socher, L. J. Li, K. Li, and L. Fei-Fei (2009). Imagenet: A large-scale hierarchical image database, *2009 IEEE Conf. on Computer Vision and Pattern Recognition*, 248–255, doi: [10.1109/CVPR.2009.5206848](https://doi.org/10.1109/CVPR.2009.5206848).
- Duan, L., C. Zhao, X. Du, and L. Zhou (2023). Machine learning-based automatic construction of earthquake catalog for reservoir areas in multiple river basins of Guizhou province, China, *Earthq. Sci.* **36**, no. 2, 132–146, [10.1016/j.eqs.2023.03.002](https://doi.org/10.1016/j.eqs.2023.03.002).
- Earp, S., A. Curtis, X. Zhang, and F. Hansteen (2020). Probabilistic neural network tomography across Grane field (North Sea) from surface wave dispersion data, *Geophys. J. Int.* **223**, no. 3, 1741–1757, doi: [10.1093/gji/ggaa328](https://doi.org/10.1093/gji/ggaa328).
- He, K., X. Zhang, S. Ren, and J. Sun (2016). Identity mappings in deep residual networks, *European Conf. on Computer Vision*, Springer International Publishing, doi: [10.1109/CVPR.2009.5206848](https://doi.org/10.1109/CVPR.2009.5206848).
- Hedjazian, N., T. Bodin, and L. Métivier (2019). An optimal transport approach to linearized inversion of receiver functions, *Geophys. J. Int.* **216**, no. 1, 130–147.
- Herrmann, R. B. (2013). Computer programs in seismology: An evolving tool for instruction and research, *Seismol. Res. Lett.* **84**, 1081–1088, doi: [10.1785/0220110096](https://doi.org/10.1785/0220110096).
- Hu, J., R. Qiu, H. Zhang, and Y. Ben-Zion (2020). Using deep learning to derive shear-wave velocity models from surface-wave dispersion data, *Seismol. Res. Lett.* **91**, no. 3, [10.1785/0220190222](https://doi.org/10.1785/0220190222).
- Julia, J., C. J. Ammon, R. B. Herrmann, and A. M. Correig (2000). Joint inversion of receiver function and surface wave dispersion observations, *Geophys. J. Int.* **143**, no. 1, 99–112, doi: [10.1046/j.1365-246x.2000.00217.x](https://doi.org/10.1046/j.1365-246x.2000.00217.x).
- Kennett, B. L. N., and E. R. Engdahl (1991). Travel times for global earthquake location and phase identification, *Geophys. J. Int.* **105**, 429–465.
- Kononenko, I. (1989). Bayesian neural networks, *Biol. Cybern.* **61**, no. 5, 361–370.
- Lakshminarayanan, B., A. Pritzel, and C. Blundell (2017). Simple and scalable predictive uncertainty estimation using deep ensembles, *Advances in Neural Information Processing Systems*, 30 pp.
- Langston, C. A. (1979). Structure under Mount Rainier, Washington, inferred from teleseismic body waves, *J. Geophys. Res.* **84**, 4749–4762, doi: [10.1029/JB084iB09p04749](https://doi.org/10.1029/JB084iB09p04749).
- Li, J., X. Song, L. Zhu, and Y. Deng (2017). Joint inversion of surface wave dispersions and receiver functions with P velocity constraints: Application to Southeastern Tibet, *J. Geophys. Res.* **122**, 7291–7310, doi: [10.1002/2017JB014135](https://doi.org/10.1002/2017JB014135).
- Li, M., X. Song, J. Li, and X. Bao (2022). Crust and upper mantle structure of East Asia from ambient noise and earthquake surface wave tomography, *Earthq. Sci.* **35**, no. 2, 71–92, doi: [10.1016/j.eqs.2022.05.004](https://doi.org/10.1016/j.eqs.2022.05.004).
- Ligorria, J. P., and C. J. Ammon (1999). Iterative deconvolution and receiver-function estimation, *Bull. Seismol. Soc. Am.* **89**, no. 5, 1395–1400, doi: [10.1785/BSSA0890051395](https://doi.org/10.1785/BSSA0890051395).
- McBrearty, I. W., A. A. Delorey, and P. A. Johnson (2019). Pairwise association of seismic arrivals with convolutional neural networks, *Seismol. Res. Lett.* **90**, no. 2A, 503–509, doi: [10.1785/0220180326](https://doi.org/10.1785/0220180326).
- Meier, U., A. Curtis, and J. Trampert (2007). Global crustal thickness from neural network inversion of surface wave data, *Geophys. J. Int.* **169**, no. 2, 706–722, doi: [10.1111/j.1365-246X.2007.03373.x](https://doi.org/10.1111/j.1365-246X.2007.03373.x).
- Mousavi, S. M., and G. C. Beroza (2022). Deep-learning seismology, *Science* **377**, no. 6607, eabm4470, doi: [10.1126/science.abm4470](https://doi.org/10.1126/science.abm4470).
- Mousavi, S. M., W. L. Ellsworth, W. Zhu, L. Y. Chuang, and G. C. Beroza (2020). Earthquake transformer—An attentive deep-learning model for simultaneous earthquake detection and phase picking, *Nat. Commun.* **11**, no. 1, 3952, doi: [10.1038/s41467-020-17591-w](https://doi.org/10.1038/s41467-020-17591-w).
- Owens, T. J., G. Zandt, and S. R. Taylor (1984). Seismic evidence for an ancient rift beneath the Cumberland plateau, Tennessee: A detailed analysis of broadband teleseismic p waveforms, *J. Geophys. Res.* **89**, no. B9, doi: [10.1029/JB089iB09p07783](https://doi.org/10.1029/JB089iB09p07783).
- Özalaybey, S., M. K. Savage, A. F. Sheehan, J. N. Louie, and J. N. Brune (1997). Shear-wave velocity structure in the northern Basin and Range province from the combined analysis of receiver functions and surface waves, *Bull. Seismol. Soc. Am.* **87**, no. 1, 183–199, doi: [10.1785/BSSA0870010183](https://doi.org/10.1785/BSSA0870010183).
- Raschka, S., and V. Mirjalili (2019). *Python Machine Learning: Machine Learning and Deep Learning with Python, Scikit-Learn, and TensorFlow 2*, Packt Publishing Ltd, Birmingham, United Kingdom.
- Ronneberger, O., P. Fischer, and T. Brox (2015). U-Net: Convolutional networks for biomedical image segmentation, *International Conference on Medical Image Computing and Computer-Assisted Intervention*, Springer International Publishing, doi: [10.1007/978-3-319-24574-4_28](https://doi.org/10.1007/978-3-319-24574-4_28).
- Ross, Z. E., and Y. Ben-Zion (2014). Automatic picking of direct P, S seismic phases and fault zone head waves, *Geophys. J. Int.* **199**, no. 1, 368–381, doi: [10.1093/gji/ggu267](https://doi.org/10.1093/gji/ggu267).
- Ross, Z. E., Y. Yue, M. A. Meier, E. Hauksson, and T. H. Heaton. (2019). PhaseLink: A deep learning approach to seismic phase association, *J. Geophys. Res.* **124**, no. 1, 856–869, doi: [10.1029/2018JB016674](https://doi.org/10.1029/2018JB016674).
- Shen, W., and M. H. Ritzwoller (2016). Crustal and uppermost mantle structure beneath the United States, *J. Geophys. Res.* **121**, no. 6, 4306–4342, doi: [10.1002/2016JB012887](https://doi.org/10.1002/2016JB012887).
- Shen, W., M. H. Ritzwoller, V. Schulte-Pelkum, and F. C. Lin (2013). Joint inversion of surface wave dispersion and receiver functions: A Bayesian Monte-Carlo approach, *Geophys. J. Int.* **192**, no. 2, 807–836, doi: [10.1093/gji/ggs050](https://doi.org/10.1093/gji/ggs050).
- Shen, Y., A. F. Sheehan, K. G. Dueker, C. Groot-Hedlinde, and H. Gilbert (1998). Mantle discontinuity structure beneath the southern East Pacific Rise from P-to-S converted phases, *Science* **280**, 1232–1235, doi: [10.1126/science.280.5367.1232](https://doi.org/10.1126/science.280.5367.1232).
- Sun, X., X. Bao, M. Xu, D. W. Eaton, X. Song, L. Wang, Z. Ding, N. Mi, D. Yu, and H. Li (2014). Crustal structure beneath SE Tibet from joint analysis of receiver functions and Rayleigh wave dispersion, *Geophys. Res. Lett.* **41**, 1479–1484, doi: [10.1002/2014GL059269](https://doi.org/10.1002/2014GL059269).
- Wang, F., X. Song, and J. Li (2022). Deep learning-based H-κ method (HkNet) for estimating crustal thickness and Vp/Vs ratio from receiver functions, *J. Geophys. Res.* doi: [10.1029/2022JB023944](https://doi.org/10.1029/2022JB023944).
- Wang, F., X. Song, and M. Li (2023). A deep-learning-based approach for seismic surface-wave dispersion inversion (SfNet) with application to the Chinese mainland, *Earthq. Sci.* **36**, no. 2, 147–168, doi: [10.1016/j.eqs.2023.02.007](https://doi.org/10.1016/j.eqs.2023.02.007).
- Wang, J., Z. Xiao, C. Liu, D. Zhao, and Z. Yao (2019). Deep-learning for picking seismic arrival times, *J. Geophys. Res.* **124**, 6612–6624, doi: [10.1029/2019JB017536](https://doi.org/10.1029/2019JB017536).

- Wang, Q., X. Song, and J. Ren (2017). Ambient noise surface wave tomography of marginal seas in east Asia, *Earth Planet. Phys.* **1**, no. 1, 13–25, doi: [10.26464/epp2017003](https://doi.org/10.26464/epp2017003).
- Wang, X., Z. Zhan, M. Zhong, P. Persaud, and R. W. Clayton (2021). Urban basin structure imaging based on dense arrays and Bayesian array-based coherent receiver functions, *J. Geophys. Res.* **9**, 126, doi: [10.1029/2021JB022279](https://doi.org/10.1029/2021JB022279).
- Wessel, P., and W. H. F. Smith (1998). New, improved version of generic mapping tools released, *Eos, Trans. Am. Geophys. Union* **79**, no. 47, 579, doi: [10.1029/98eo00426](https://doi.org/10.1029/98eo00426).
- Woo, S., J. Park, J. Y. Lee, and I. S. Kweon (2018). Cbam: Convolutional block attention module, *Proc. of the European Conf. on Computer Vision (ECCV)*, 3–19.
- Xu, Z., X. Song, and L. Zhu (2013). Crustal and uppermost mantle S velocity structure under Hi-CLIMB seismic array in central Tibetan Plateau from joint inversion of surface wave dispersion and receiver function data, *Tectonophysics* **584**, 209–220, doi: [10.1016/j.tecto.2012.08.024](https://doi.org/10.1016/j.tecto.2012.08.024).
- Yang, F., and J. Ma (2019). Deep-learning inversion: A next generation seismic velocity-model building method, *Geophysics* doi: [10.1190/geo2018-0249.1](https://doi.org/10.1190/geo2018-0249.1).
- Yang, T., D. Cao, N. Du, R. Cui, F. Nan, Y. Xu, and C. Liang (2022). Prediction of shear-wave velocity using receiver functions based on the deep learning method, *Chin. J. Geophys.* **65**, no. 1, 214–226, doi: [10.6038/cjg2022P0025](https://doi.org/10.6038/cjg2022P0025).
- Ying, L., H. Zhang, H. Fang, H. Yao, and G. Ji (2018). Ambient noise tomography of three-dimensional near-surface shear-wave velocity structure around the hydraulic fracturing site using surface microseismic monitoring array, *J. Appl. Geophys.* **159**, 209–217, doi: [10.1016/j.jappgeo.2018.08.009](https://doi.org/10.1016/j.jappgeo.2018.08.009).
- Yuan, X., J. Ni, R. Kind, J. Mechie, and E. Sandvol (1997). Lithospheric and upper mantle structure of southern Tibet from a seismological passive source experiment, *J. Geophys. Res.* **102**, 27,491–27,500, doi: [10.1029/97JB02379](https://doi.org/10.1029/97JB02379).
- Zhang, P., and H. Yao (2017). Stepwise joint inversion of surface wave dispersion, Rayleigh wave ZH ratio, and receiver function data for 1D crustal shear wave velocity structure, *Earthq. Sci.* **30**, 229–238, doi: [10.1007/s11589-017-0197-0](https://doi.org/10.1007/s11589-017-0197-0).
- Zhang, L., L. Fang, W. Wang, and Z. Lv (2020). Seismic phase picking in China seismic array using a deep convolutional neuron network, *Earthq. Sci.* **33**, no. 2, 72–81, doi: [10.29382/eqs-2020-0072-03](https://doi.org/10.29382/eqs-2020-0072-03).
- Zhao, Y., Y. Duan, Z. Yang, and Z. Gao (2020). S-wave velocity structure in Tangshan earthquake region and its adjacent areas from joint inversion of receiver functions and surface wave dispersion, *Earthq. Sci.* **30**, no. 1, 42–52, doi: [10.29382/eqs-2020-0042-05](https://doi.org/10.29382/eqs-2020-0042-05).
- Zhou, Y., G. Nolet, F. A. Dahlen, and G. Laske (2006). Global upper-mantle structure from finite-frequency surface-wave tomography, *J. Geophys. Res.* **111**, no. B4, [10.1029/2005JB003677](https://doi.org/10.1029/2005JB003677).
- Zhou, Y., H. Yue, L. Fang, S. Zhou, L. Zhao, and A. Ghosh (2021). An earthquake detection and location architecture for continuous seismograms: Phase picking, association, location, and matched filter (PALM), *Seismol. Res. Lett.* doi: [10.1785/0220210111](https://doi.org/10.1785/0220210111).
- Zhu, L., and H. Kanamori (2000). Moho depth variation in southern California from teleseismic receiver functions, *J. Geophys. Res.* **105**, no. B2, 2969–2980, doi: [10.1029/1999JB900322](https://doi.org/10.1029/1999JB900322).
- Zhu, J., Z. Li, and L. Fang (2023). USTC-Pickers: A unified set of seismic phase pickers transfer learned for China, *Earthq. Sci.* **36**, no. 2, 95–112, doi: [10.1016/j.eqs.2023.03.001](https://doi.org/10.1016/j.eqs.2023.03.001).

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